Stochastic Interdependent Volatility-Adaptive Signal Generation

Context:

In a global foreign exchange market consisting of 28 currency pairs, volatility interdependencies exhibit non-linear stochastic behaviors due to the influence of macroeconomic shocks, geopolitical events, and liquidity variations. The aim is to formulate a **stochastic signal generation framework** that leverages **PCA-enhanced feature extraction**, **Graph Neural Networks (GNNs)** for dynamic modeling of interdependencies, and **entropy-based thresholding**. The problem requires solving for the dynamic thresholds under constraints imposed by the stochastic differential equations governing volatility relationships, while ensuring market adaptiveness, higher precision, and robustness.

Notations and Variables:

1. $C_i, i = 1, \ldots, 28$: Currency pairs.
2. $\sigma_i(t)$: Volatility of C_i at time t, modeled as a stochastic process.
3. ρ_{ij}: Correlation between volatilities of C_i and C_j, dynamically evolving.
4. $\Theta_i(t)$: Dynamic threshold for trading signal generation for C_i.
5. $GNN(\cdot)$: GNN operator capturing interdependencies among PCA-reduced features.
6. $\mathrm{H}_i(t)$: Entropy of the GNN output probabilities for C_i.

Problem Formulation:

Volatility Dynamics:

The volatility of each currency pair follows a **multivariate stochastic volatility process** influenced by common market factors:

$$d\sigma_i(t) = \mu_i(t, \sigma_i)dt + \sum_{j=1}^{28} \rho_{ij}(t, \sigma_i, \sigma_j)\sigma_j dW_j(t),$$

where:

- $\mu_i(t, \sigma_i)$: Drift term dependent on the mean-reversion of $\sigma_i(t)$.
- $\rho_{ij}(t, \sigma_i, \sigma_j)$: Time-varying correlation term, dependent on PCA-reduced feature distances and market liquidity.
- $W_j(t)$: Independent Wiener processes.

GNN-Enhanced Dependencies:

The PCA-reduced feature matrix $\mathbf{R}_i(t)$ for each C_i evolves as:

$$\mathbf{R}_i(t) = \sum_{k=1}^{m} \lambda_k \mathbf{v}_k + \eta_i(t),$$

where:

- λ_k: Eigenvalues of the kernel matrix.

- $\mathbf{v}_k$: Eigenvectors.
- $\eta_i(t)$: Noise modeled as a correlated stochastic process.

The adjacency matrix for the GNN at time t is derived using the Euclidean distance between the reduced features:

$$A_{ij}(t) = \exp\left(-\frac{\|\mathbf{R}_i(t) - \mathbf{R}_j(t)\|^2}{2\sigma_A^2}\right),$$

where σ_A is a sensitivity parameter.

Entropy-Based Threshold Dynamics:

The entropy of the GNN output probabilities $\mathrm{H}_i(t)$ for C_i is:

$$\mathrm{H}_i(t) = -\sum_{k=1}^{3} p_i^{(k)}(t) \log(p_i^{(k)}(t)),$$

where $p_i^{(k)}(t)$ is the probability of action k (Buy, Sell, Hold).

The dynamic threshold $\Theta_i(t)$ incorporates:

1. **Volatility change rate**:

$$\Delta\sigma_i(t) = \sigma_i(t) - \sigma_i(t - \Delta t).$$

2. **PCA variance contribution**:

$$\Theta_{\mathrm{PCA}}(t) = \frac{\sum_{k=1}^{m} \lambda_k}{\sum_{k=1}^{N} \lambda_k}.$$

3. **Entropy of GNN outputs**:

$$\Theta_{\mathrm{Entropy}}(t) = 1 - \mathrm{H}_i(t).$$

The combined threshold is:

$$\Theta_i(t) = \Delta\sigma_i(t) + \alpha \cdot \Theta_{\mathrm{PCA}}(t) \cdot \Theta_{\mathrm{Entropy}}(t),$$

where α is a scaling factor.

Signal Generation:

The trading signal for C_i is defined as:

$$S_i(t) = \mathrm{I}\left(\text{Signal Strength} > \Theta_i(t)\right),$$

where Signal Strength is the difference between GNN probabilities for Buy and Sell actions.

Problem Objective:

1. **Optimize Trading Signals**:
   - Maximize the Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{\mathrm{E}[R]}{\mathrm{StdDev}[R]},$$

where R is the portfolio return.

2. **Ensure Robustness**:
   - Minimize signal variance due to noise in $\mathrm{H}_i(t)$ and $\rho_{ij}(t)$.
3. **Dynamic Adaptability**:
   - The thresholds $\Theta_i(t)$ should dynamically respond to:
     - Changes in market regime.
     - Stochastic volatility patterns.
4. **Multi-Dimensional Complexity**:
   - Solve for $\Theta_i(t)$ under the constraints of the stochastic differential equations governing $\sigma_i(t)$ and $\rho_{ij}(t)$.

Constraints:

- **Dimensionality Reduction**: PCA variance contribution should explain at least 90% of the total variance.
- **GNN Convergence**: The GNN output probabilities must converge within a tolerance ϵ over t.
- **Threshold Stability**: The entropy-weighted thresholds should not fluctuate beyond 10% across consecutive time intervals.

Task: Derive the analytical form of $\Theta_i(t)$ satisfying the above objectives and constraints, and implement a computational framework to solve the stochastic differential equations governing the strategy.